

# Model Independent $Z'$ Constraints at Future $e^+e^-$ Colliders

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## Abstract

Model independent constraints on the mass of extra neutral gauge bosons and their couplings to charged leptons are given for LEP II and a 500 GeV  $e^+e^-$  collider. Analytical exclusion limits are derived in the Born approximation. The  $Z'$  limits obtained with radiative corrections are always worse than those calculated at the Born level. Polarized beams are only useful for degrees of polarization essentially larger than 50%. Known discovery limits on extra  $Z$  bosons predicted by popular  $Z'$  models are reproduced as special cases. The  $Z'$  constraints are compared to those predicted by four fermion contact interactions.

## 1 Introduction

The Standard Model (SM) gives a very successful description of the present high energy physics data. Its predictions are proven at the LEP I energy range with high precision being sensitive to physics at the one loop level. However, there is a common belief that the strong and electroweak interactions of the SM should have a common origin. Their unification is often done in a large gauge group at higher energies. The embedding of the SM in a gauge group larger than SU(5) typically predicts extra neutral gauge bosons denoted by a  $Z'$  here. The search for such a particle is an important task of any present and future collider.

Up to now no extra neutral gauge boson is found. The consistency of experimental data with the SM is usually interpreted in terms of  $Z'$  mass limits for specific  $Z'$  models as, for example, the  $E_6$  GUT [1, 2], the Left Right Symmetric Model (LRM) [2, 3] or the Sequential Standard Model (SSM). Although these limits give an important feeling about the predictive power of an experiment, the full information about a  $Z'$  can only be obtained by a model independent  $Z'$  analysis of the experimental data. Such a general analysis is possible for the leptonic couplings of the  $Z'$  due to the large number of pure leptonic observables at  $e^+e^-$  colliders. The hadronic observables are sensitive to the leptonic couplings of the  $Z'$  too. Therefore, the measurement of the  $Z'$  couplings to quarks in  $e^+e^-$  collisions makes sense only, if non-zero  $Z'$  couplings to charged leptons are found.

At hadron colliders, the squares of the  $Z'$  couplings to quarks and leptons can be measured [4]. Although the sign of the couplings cannot be defined there, even a  $Z'$  with zero couplings to charged leptons could be detected via the search for rare processes. As we will show later,  $e^+e^-$  colliders allow a discrimination between different signs of  $Z'$  couplings. Hadron and  $e^+e^-$  colliders are complementary in  $Z'$  search.

In this paper, we make a model independent  $Z'$  analysis and show, how the different leptonic observables compete in constraining the leptonic  $Z'$  couplings. Known limits for specific  $Z'$  models and for four fermion contact interactions are reproduced as special cases. At the Born level, approximate analytical formulae are obtained. They make the analysis and its dependence on the experimental errors transparent. They are not very different from the final result which contains all needed radiative corrections. Radiative corrections have to be included because a  $Z'$  gives no events with special signature but shows up by (probably small) deviations of observables from their SM predictions.

Taking into account present experimental limits on the  $Z'$  mass [5] and the expected improvements in near future [6], LEP II and the Next Linear  $e^+e^-$  Collider with a c.m. energy of 500 GeV (NLC) will probably operate well below a  $Z'$  peak. Further, the  $ZZ'$  mixing angle is known to be small [7] and can be set to zero in our analysis.

Larger gauge groups predict not only extra gauge bosons but also additional fermions. Their effects are described, for example, in [1, 8] and will not be considered here.

In section 2, we give the basic notations of a model independent  $Z'$  description and show the connections to some popular  $Z'$  models and to four fermion contact interactions. Section 3 contains the  $Z'$  analysis at the Born level leading to approximate analytical formulae. In section 4, we make a model independent  $Z'$  analysis for LEP II and the NLC with all needed radiative corrections. Mass limits for popular  $Z'$  models and for models with contact interactions are obtained as special cases. In the case of a positive  $Z'$  signal, the interpretation in terms of the  $Z'$  mass or bounds on model parameters is demonstrated.

## 2 Model Independent $Z'$ Description

Extra neutral gauge bosons lead to additional neutral current interactions with SM fermions

$$\mathcal{L} = eA_\mu J_\gamma^\mu + g_1 Z_\mu J_Z^\mu + g_2 Z'_\mu J_{Z'}^\mu. \quad (1)$$

We assume the gauge group  $SU(2)_L \times U(1)_Y \times U'(1)$  at low energies, where  $SU(2)_L \times U(1)_Y$  belongs to the SM and  $U'(1)$  to the  $Z'$ . The amplitude of fermion pair production induced by the new interaction of the  $Z'$  is

$$\begin{aligned} \mathcal{M}(Z') &= \frac{g_2^2}{s - m_{Z'}^2} \bar{u}_e \gamma_\beta (\gamma_5 a'_e + v'_e) u_e \bar{u}_f \gamma^\beta (\gamma_5 a'_f + v'_f) u_f \\ &= -\frac{4\pi}{s} \left[ \bar{u}_e \gamma_\beta (\gamma_5 a_e^N + v_e^N) u_e \bar{u}_f \gamma^\beta (\gamma_5 a_f^N + v_f^N) u_f \right] \end{aligned} \quad (2)$$

$$\text{with } a_f^N = a'_f \sqrt{\frac{g_2^2}{4\pi} \frac{s}{m_{Z'}^2 - s}}, \quad v_f^N = v'_f \sqrt{\frac{g_2^2}{4\pi} \frac{s}{m_{Z'}^2 - s}} \quad \text{and} \quad m_{Z'}^2 = M_{Z'}^2 - i\Gamma_{Z'} M_{Z'} \quad (3)$$

at the Born level. The minus sign reflects the destructive interference of the  $Z'$  contributions with the photon and  $Z$  exchange below the  $Z'$  peak. The normalized couplings to leptons  $a_l^N$

and  $v_l^N$  will be restricted by future data. Far away from resonances, the exact formulae (3) for  $a_f^N$  and  $v_f^N$  can be approximated,

$$a_f^N \approx a'_f \sqrt{\frac{g_2^2}{4\pi}} \frac{\sqrt{s}}{M_{Z'}}, \quad v_f^N \approx v'_f \sqrt{\frac{g_2^2}{4\pi}} \frac{\sqrt{s}}{M_{Z'}}. \quad (4)$$

In this case, the lepton pair production by a  $Z'$  can be described by effective four fermion contact interactions [9] with  $\Lambda = M_{Z'}$

$$\begin{aligned} \mathcal{M}_{ell} = \frac{g^2}{\Lambda^2} & \left( \eta_{LL} \bar{u}_{e,L} \gamma_\beta u_{e,L} \bar{u}_{l,L} \gamma^\beta u_{l,L} + \eta_{RR} \bar{u}_{e,R} \gamma_\beta u_{e,R} \bar{u}_{l,R} \gamma^\beta u_{l,R} \right. \\ & \left. + \eta_{RL} \bar{u}_{e,R} \gamma_\beta u_{e,R} \bar{u}_{l,L} \gamma^\beta u_{l,L} + \eta_{LR} \bar{u}_{e,L} \gamma_\beta u_{e,L} \bar{u}_{l,R} \gamma^\beta u_{l,R} \right). \end{aligned} \quad (5)$$

Assuming lepton universality and setting  $g^2/(4\pi) = 1$  by tradition, the constants of contact interactions can be expressed by the  $Z'$  lepton couplings

$$\eta_{LL} = (v_l^N - a_l^N)^2, \quad \eta_{RR} = (v_l^N + a_l^N)^2, \quad \eta_{RL} = \eta_{LR} = (v_l^N)^2 - (a_l^N)^2 = \pm \sqrt{\eta_{LL} \eta_{RR}}. \quad (6)$$

A general  $Z'$  analysis is therefore equivalent to an analysis in terms of contact interactions with any positive  $\eta_{LL}$  and  $\eta_{RR}$  taking the appropriate values for  $\eta_{LR}$  and  $\eta_{RL}$ . Experimental limits for representative cases of contact interactions are obtained from LEP data [10] and for future  $e^+e^-$  colliders [11]. A complete analysis of the parameter space of contact interactions has not been done.

**Fig. 1:** The normalized couplings  $(a_l^N, v_l^N)$  of the  $Z'$  to charged leptons for popular  $Z'$  models with  $M_{Z'} = 4\sqrt{s}$ . The ranges of the  $E_6$  and LR model are indicated as well as the special cases  $\psi$ ,  $\chi$ ,  $\eta$  and LR. The abbreviations AA, LL, VV, and RR correspond to contact interactions [11] with  $\Lambda = 100\sqrt{s}$ . The numbers indicate different  $Z_\chi$  masses in units of  $\sqrt{s}$ .

Fig. 1 shows some popular  $Z'$  models in terms of the generalized couplings  $a_l^N$  and  $v_l^N$ . The  $E_6$  GUT and the LRM correspond to lines parametrized by  $\cos \beta$  [1] and  $\alpha_{LR}$  [3]

$$J_{Z'}^\mu = J_\chi^\mu \cos \beta + J_\psi^\mu \sin \beta, \quad J_{Z'}^\mu = \alpha_{LR} J_{3R}^\mu - \frac{1}{2\alpha_{LR}} J_{B-L}^\mu. \quad (7)$$

Some completely specified cases are marked by  $\blacklozenge$ . The SSM, where the  $Z'$  boson has the same couplings as the SM  $Z$  boson, is also shown for comparison. Different  $Z'$  masses correspond to different points on a straight line in the  $(a_l^N, v_l^N)$  plane. A measurement of the  $Z'$  mass at hadron colliders would transform Fig. 1 into constraints to the absolute values of the coupling constants.

### 3 Model Independent Analysis at the Born level

At an  $e^+e^-$  collider, the following leptonic observables can be measured [12]

$$\sigma_t, \quad A_{FB}, \quad A_{pol}, \quad A_{pol}^{FB}, \quad A_{LR}, \quad (8)$$

where  $A_{pol}$ ,  $A_{pol}^{FB}$  are the polarization asymmetries of  $\tau$  leptons in the final state and  $\sigma_t$ ,  $A_{FB}$  and  $A_{LR}$  are obtained for the production of charged lepton pairs with the  $t$  channel for Bhabha scattering subtracted. The comparison of future measurements of these observables with its SM predictions lead to constraints on  $a_l^N$  and  $v_l^N$ . The obtained limits are sensitive to the expected experimental errors and radiative corrections.

As in [13], we assume a 1% systematic error due to the luminosity uncertainty and 0.5% due to the event selection of leptons. The systematic error of the forward backward asymmetry is assumed to be negligible and that of the left right polarization asymmetry to be  $\Delta A_{LR} = 0.3\%$ .

As statistical errors for  $N$  detected events we take:

$$\frac{\Delta\sigma_t}{\sigma_t} = \frac{1}{\sqrt{N}}, \quad \Delta A_{FB} = \Delta A_{pol} = \Delta A = \sqrt{\frac{1 - A^2}{N}}, \quad \Delta A_{LR} = \sqrt{\frac{1 - (PA_{LR})^2}{NP^2}}. \quad (9)$$

$P$  is the degree of polarization set to one here. We assume that for 75% of the  $\tau$  events the final state polarization can be measured [12]. We took an integrated luminosity  $L_{int} = 0.5 fb^{-1}$  for LEP II and  $L_{int} = 20 fb^{-1}$  for the NLC.

The systematic and statistical errors have then been combined quadratically. For completeness, we list the combined errors for every of the considered observables

$$\begin{aligned} \frac{\Delta\sigma_t}{\sigma_t} &= 1.6\%, \quad \Delta A_{FB} = 1.1\%, \quad \Delta A_{pol} = \Delta A_{pol}^{FB} = 2.8\%, \quad \Delta A_{LR} = 1.4\%, \quad \text{LEP II} \\ \frac{\Delta\sigma_t}{\sigma_t} &= 1.3\%, \quad \Delta A_{FB} = 0.5\%, \quad \Delta A_{pol} = \Delta A_{pol}^{FB} = 1.2\%, \quad \Delta A_{LR} = 0.7\%, \quad \text{NLC.} \end{aligned} \quad (10)$$

In theories with lepton universality, neglecting the difference of lepton masses, three observables are related at the Born level

$$A_{LR} = A_{pol} = \frac{4}{3} A_{pol}^{FB}. \quad (11)$$

A violation of these relations would indicate that there exist new physics beyond the SM which are not due to extra  $Z$  bosons.

Under the assumption that both asymmetries have the same experimental errors,  $A_{pol}^{FB}$  gives always worse  $Z'$  constraints than  $A_{pol}$ . Therefore, it is not considered in the further analysis. Among  $A_{LR}$  and  $A_{pol}$ , the observable with the smaller experimental error gives better  $Z'$  constraints.  $A_{LR}$  has about four times higher event rates being sensitive to all leptons. However, equation (9) shows that its predictive power is reduced by degrees of

polarization smaller than 100%. With degrees of polarizations of 50% or less, measurements of  $A_{LR}$  add no new information to the  $Z'$  search. Therefore, an upgrade of the luminosity by a factor 2 to 3 should be preferred compared to polarized beams because it reduces the error of all observables. In case of a non-zero  $Z'$  signal, quark pair production by polarized beams would help to determine the  $Z'$ -quark couplings as far as the final quark polarization is not measurable.

We now demonstrate the model independent  $Z'$  analysis at the Born level. The result are approximate analytical formulae making transparent how the model independent  $Z'$  constraints arise from different observables.

The observable  $\sigma_t$  can “see” a signal of a  $Z'$ , if

$$\chi^2 = \left( \frac{\Delta^{Z'} \sigma_t}{\Delta \sigma_t} \right)^2 \geq F_\chi^2. \quad (12)$$

$\Delta \sigma_t$  is the combined experimental error as given in (10),  $F_\chi^2$  is a number depending on the confidence level and the details of the analysis and  $\Delta^{Z'} \sigma_t$  is the deviation of  $\sigma_t$  from its SM prediction  $\sigma_t^{SM}$  due to a  $Z'$ .

In this section, we will neglect all imaginary parts of the  $Z$  and  $Z'$  propagators,

$$\chi_Z = \frac{\sqrt{2} G_\mu M_Z^2}{4\pi\alpha} \frac{s}{s - M_Z^2 + i M_Z \Gamma_Z} \approx \frac{\sqrt{2} G_\mu M_Z^2}{4\pi\alpha} \frac{s}{s - M_Z^2}. \quad (13)$$

Neglecting the quadratic  $Z'$  contributions and taking into account that the leptonic vector coupling of the SM  $Z$  boson  $v_l$  is small against 1 and the axial vector coupling  $a_l = -1/2$ , one obtains

$$\Delta^{Z'} \sigma_t \approx \frac{4\pi\alpha}{3s} 2 \left[ (v_l^N)^2 + \chi_Z a_l^2 (a_l^N)^2 \right]. \quad (14)$$

The first contribution is due to the  $\gamma Z'$  interference, the second is due to the  $ZZ'$  interference. Equation (14) together with (12) give the constraint on  $a_l^N$  and  $v_l^N$  by  $\sigma_t$ ,

$$\left( \frac{v_l^N}{H_t} \right)^2 + \left( \frac{a_l^N}{H_t} \right)^2 \frac{\chi_Z}{4} \geq 1, \quad H_t = \sqrt{\frac{F_\chi \alpha}{2} \frac{\sigma_t^{SM}}{\sigma_t^{QED}} \frac{\Delta \sigma_t}{\sigma_t^{SM}}}. \quad (15)$$

In (15),  $\sigma_t^{QED} = \frac{4\pi\alpha^2}{3s}$  is the QED cross section. We conclude from (15) that the observable  $\sigma_t$  cannot exclude an ellipse of  $(a_l^N, v_l^N)$  around the point (0,0).

Similar considerations can be done for the observables  $A_{FB}$  and  $A_{LR}$ . They lead to the following exclusion regions

$$\left| \left( \frac{v_l^N}{H_{FB}} \right)^2 - \left( \frac{a_l^N}{H_{FB}} \right)^2 \frac{(3 - A_{FB}^{SM} \chi_Z) \frac{1}{4}}{A_{FB}^{SM} - \frac{3}{16} \chi_Z} \right| \geq 1, \quad H_{FB} = \sqrt{\frac{F_\chi \alpha}{2} \frac{\sigma_t^{SM}}{\sigma_t^{QED}} \Delta A_{FB}} \left[ A_{FB} - \frac{3}{16} \chi_Z \right]^{-1/2}, \quad (16)$$

$$\left| \left( \frac{v_l^N}{H_{LR}} \right) \left( \frac{a_l^N}{H_{LR}} \right) \right| \geq 1, \quad H_{LR} = \sqrt{\frac{F_\chi \alpha}{2} \frac{\sigma_t^{SM}}{\sigma_t^{QED}} \Delta A_{LR}} \left[ 1 + \frac{1}{4} \chi_Z \right]^{-1/2}. \quad (17)$$

In the last two equations,  $A_{FB}^{SM}$  and  $A_{LR}^{SM}$  are the SM predictions for the corresponding observables. The two considered asymmetries  $A_{FB}$  and  $A_{LR}$  cannot exclude the area between hyperbolas. We see that the three considered observables are complementary in excluding

$Z'$  contributions. The equations (15) - (17) reflect all qualitative features of the results with radiative corrections to be discussed in the next section.

$H_t$ ,  $H_{FB}$  and  $H_{LR}$  are a measure of the sensitivity to  $Z'$  effects. They are all proportional to the square root of the experimental error. Taking into account only the statistical error, we obtain a simple scaling law of the  $Z'$  exclusion limits with the integrated luminosity  $L_{int}$  and the c.m. energy  $\sqrt{s}$ ,

$$a_i^N, v_i^N \sim H_t, H_{FB}, H_{LR} \sim N^{-1/4} \sim (L_{int}/s)^{-1/4}. \quad (18)$$

Together with (4), we get for the scaling of the  $Z'$  mass limit

$$M_{Z'}^{lim} \sim \sqrt{s}/a_i^N, \sqrt{s}/v_i^N \sim (L_{int}s)^{1/4}. \quad (19)$$

This agrees with the results of [11] and [14].

In case of a positive  $Z'$  signal  $(\bar{a}_i^N, \bar{v}_i^N) \neq (0, 0)$ , experimental constraints on  $(a_i^N, v_i^N)$  could be obtained by modifications of the above deviation. In contrast to the previous discussion, the size of the area in the  $(a_i^N, v_i^N)$  plane which cannot be resolved is now proportional to  $M_{Z'}$  and centered around the point  $(\bar{a}_i^N, \bar{v}_i^N)$ . Suppose, that the  $Z'$  couplings are proportional to some model parameter  $\alpha_{Z'}$  and that the  $Z'$  mass is  $M_{Z'}$ . Suppose further, that  $\alpha_{Z'}$  and  $M_{Z'}$  can be measured with errors  $\Delta\alpha_{Z'}$  and  $\Delta M_{Z'}$ . Then, the scaling of these errors with the  $Z'$  mass is

$$M_{Z'} \rightarrow c M_{Z'} \implies \Delta M_{Z'} \rightarrow c^3 \Delta M_{Z'} \text{ and } \Delta\alpha_{Z'} \rightarrow c^2 \Delta\alpha_{Z'}. \quad (20)$$

The scaling law discussed above is only valid for a  $Z'$  mass larger than the c.m. energy and smaller than  $M_{Z'}^{lim}$ . Equation (20) agrees well with our numerical results and those of [13] and [14].

The squares of the couplings  $(a_i^N)^2, (v_i^N)^2$  are expected to have a gaussian error because they are proportional to parts of cross sections. These squares don't feel the sign of  $a_i^N$  as well as the equations (15) and (16). However, for a non-zero  $Z'$  signal, the observables  $A_{LR}$ ,  $A_{pol}$  and  $A_{pol}^{FB}$  lead to deviations from their SM predictions which have a different sign for different signs of  $a_i^N$ . Furthermore, the exact deviation of (15) - (17) shows also a weak dependence on the sign of  $a_i^N$  due to terms of the order  $O(v_i^N/a_i^N)$  neglected in the approach above. Hence, in a model independent  $Z'$  search, the simple couplings and not its squares should be analyzed although their errors are not distributed gaussian.

## 4 The Analysis including Radiative Corrections

All calculations are done using the code ZCAMEL [15], which contains the  $O(\alpha)$  QED corrections with soft photon exponentiation for the process  $e^+e^- \rightarrow (\gamma, Z, Z') \rightarrow f^+f^-$ . A cut  $\Delta$  on the energy of the radiated photon  $\Delta = 0.7 > E_\gamma/E_{beam}$  has been applied to remove the radiative tail and the potentially dangerous background [13]. The analytical formulae of the QED corrections used here are given in [16]. The SM electroweak corrections must not be considered because they cancel in the *deviation* of an observable from its SM value. The main effect of the radiative corrections is a relaxation of the  $Z'$  limits of roughly 10% compared to the Born result.

The constraints to  $a_i^N$  and  $v_i^N$  are shown for LEP II in Fig. 2a and for the NLC in Fig. 2b. All (one sided) exclusion limits are at the 95% c.l., which corresponds to  $\chi^2 = F_\chi^2 = 2.69$  for one degree of freedom [17]. They should be compared with the approximate analytical results

of (15) - (17). In general, the constraints to  $a_i^N$  and  $v_i^N$  are worse at LEP II because it has larger errors compared to the NLC, see (10).

**Fig. 2a:** The areas in the  $(a_i^N, v_i^N)$  plane excluded by different observables at LEP II with 95% confidence. The allowed range always contains the point  $(0,0)$ .

**Fig. 2b:** The same as Fig. 2a but for the NLC. The numbers indicate the  $Z_\chi$  mass in units of  $\sqrt{s}$ .

The Figures 2a and 2b are model independent. They can be used to obtain mass limits for particular  $Z'$  models. Consider the  $Z_\chi$  arising in the breaking chain of the  $E_6$  GUT [1] at Fig. 2b. We see that  $\sigma_t$  and  $A_{LR}$  give mass limits of  $M_{Z'} > 2.2 TeV$  at the NLC, while the constraint from  $A_{FB}$  is  $M_{Z'} > 1 TeV$ . The corresponding intersection point is outside of Fig. 2b. This is in agreement with the mass limits obtained for the  $E_6$  GUT under the same experimental conditions in [13]. The mass limits for any other  $Z'$  model from LEP II and the NLC could be obtained from Fig. 2a and Fig. 2b in a similar way. The limits on four fermion contact interactions agree with [11] if one takes into account the differences between the two analyses.

For  $M_{Z'} \approx \frac{2}{3}M_{Z'}^{lim}$ , any point  $(a_i^N, v_i^N)$  is covered by at least two observables, where  $M_{Z'}^{lim}$  is the discovery limit for the same  $Z'$  model, see Figs. 2. The leptonic couplings of the  $Z'$  can then be measured as demonstrated in Figs. 3. Note the difference in the scale compared to Figs. 1 and 2. The area of  $(a_i^N, v_i^N)$  allowed with 95% confidence by one observable alone is defined by  $\chi^2 = 4$ . The combined allowed area around the  $\clubsuit$  of all three observables is obtained taking  $\chi^2 = 7.7$ . A more concrete interpretation of this area demands specifications of  $M_{Z'}$  or the  $Z'$  model. Fixing  $Z' = Z_\chi$ , Fig. 3b gives the errors of the measurement of  $M_{Z'}$ . Fixing  $M_{Z'} = 3\sqrt{s} = 1.5 TeV$ , Fig. 3b gives the errors of the measurement of model parameters as for  $\cos\beta$  in the  $E_6$  model,  $\alpha_{LR}$  in the LRM or areas of confusion between these models.

**Fig. 3a:** The area  $(a_i^N, v_i^N)$  allowed by  $\sigma_t$ ,  $A_{FB}$  and  $A_{LR}$  separately with 95% confidence.  $Z' = Z_\chi$  with  $M(Z_\chi) = 3\sqrt{s}$  was chosen and marked by  $\clubsuit$ . The combined area allowed by all three observables is indicated by the thick dotted line.

**Fig. 3b:** Demonstration of the measurement of the  $Z'$  mass and its couplings to charged leptons at the NLC. The thick dotted line around the 3 is copied from Fig. 3a. The thin dotted lines around the 2 (4) correspond to  $M_{Z'} = 2$  (4) $\sqrt{s}$ . For all other lines see Fig. 1.

If future measurements are inconsistent with the existence of a  $Z'$ , they should be described in the more general scheme of four fermion contact interactions. In contrast to  $Z'$  models, negative signs of  $\eta_{LL}$  and  $\eta_{RR}$  are allowed and  $\eta_{LR}$  and  $\eta_{RL}$  are independent of  $\eta_{LL}$  and  $\eta_{RR}$ . An inconsistency of measurements with  $Z'$  theories could arise due to the violation of (11) or due to a zero allowed area for  $(a_i^N, v_i^N)$  after combining all observables.

Consider the case of a very weakly coupling  $Z'$ . The resonance curve of such a  $Z'$  is very narrow and high. It can be detected only very near its resonance. The observables near a resonance cannot be described by four fermion contact interactions. Radiative corrections become much more important making the Figs. 2 and 3 dependent on the  $Z'$  mass. However, we have proven that even for the extreme case  $M_{Z'} = 505 \text{ GeV} = 1.01\sqrt{s}$  at the NLC, the effect of radiative corrections is only a relaxation of the limits on  $a_i^N$  and  $v_i^N$  by 30% leaving Figs. 2b and 3 qualitatively unchanged. Eventually, a weakly coupling  $Z'$  would never be missed at energies above the  $Z'$  peak, because its radiative tail is proportional to  $M_{Z'}/\Gamma_{Z'}$ .

To summarize, a model independent  $Z'$  analysis is made for future  $e^+e^-$  colliders. Numerical examples are given for LEP II and a 500 GeV  $e^+e^-$  collider. Assuming lepton universality, all available leptonic observables except Bhabha scattering are considered. Approximate analytical formulae for  $Z'$  exclusion limits and scaling laws are obtained. All needed radiative corrections are included in the analysis and their effects on  $Z'$  constraints are discussed. It is shown, how the different observables give complementary constraints to  $Z'$  physics. Polarized beams add useful information to the  $Z'$  search only for degrees of polarization essentially larger than 50%. In case of a positive  $Z'$  signal, it is demonstrated, under which conditions the  $Z'$  couplings to leptons and the  $Z'$  mass can be measured. Connections to four fermion contact interactions are shown as well as to popular  $Z'$  models.



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